Improving Assessment Equity in Mass Appraisal Models

BY CARMELA QUINTOS, PH.D.

Assessment equity is a primary concern for municipalities that levy ad valorem taxes. Property taxes are a popular source of revenue for funding local services, requiring that assessment levels be fair for properties across price levels (vertical equity) and across properties with similar characteristics (horizontal equity). The standard measures for horizontal and vertical equity published by the International Association of Assessing Officers (IAAO) are the coefficient of dispersion (COD) and the price-related differential (PRD), respectively.

Modeling heterogeneity is essential to achieving a COD and a PRD that conform to IAAO standards. Failure to model the variation in price by correctly specifying the regression equation with relevant property characteristics can be reflected in residuals that are heteroskedastic. Heteroskedasticity, which means that the variance of the residuals (or pricing error) is not constant across certain property characteristics, is by definition a failure of horizontal equity and indicative of a high COD.

Similarly, the PRD is known to be affected by unexplained heterogeneity (i.e., heteroskedastic residuals). Regression methods, which average effects, tend toward regressive assessments when heterogeneity is not modeled. Denne (2011), Gloudemans (2011), Jensen (2009), and Hodge et al. (2013) discuss the relation of heteroskedasticity and the regressive bias of the PRD. (The IAAO range for the COD is 5–15 percent for single-family homes, and the range for the PRD is 0.98–1.03. To test whether these ranges are satisfied at a significance level, bootstrap methods are used to construct confidence intervals. When heteroskedasticity is present, the standard bootstrap method fails (see Cribari-Neto and Zarkos [2004] for an alternative approach).

Several methods for dealing with heterogeneity are geographic information systems (GIS) methods, which allow coefficients to vary across location

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(GWR, spatial interpolation), fixed and random effect regression models, and Kalman filter models. Quantile regression models have been used to diagnose heterogeneity and assessment equity (McMillen 2011) but have not been used in the context of mass appraisal valuation. One problem is that unsold parcels (whose prices are unknown) need to be assigned into the appropriate quantile model. Furthermore, parcel assignment into models must maintain some consistency; for example, similar properties in the same block, area, or cluster should be assigned to the same model.

This article shows how quantile regression models are used in the mass appraisal of single-family homes in New York City, particularly in the borough of Brooklyn, where heterogeneity is most acute. The method is a two-step process. First, a spatial lag specification addresses heterogeneity across property characteristics, which improves the COD and enables statistical inference using *t* values. Second, the spatial lag specification is run at different percentiles using quantile regressions. This second step reduces the distortion in the PRD by reducing the correlation between assessment ratios and price levels.

To extend the procedure to unsold parcels for mass appraisal, the assignment of parcels into percentiles is handled by using spatially interpolated prices. In other words, a price surface is first estimated by using prices of *sold* parcels, then the prices of *unsold* parcels are inferred from this surface. Spatial estimation and interpolation of the price surface are done by block coordinates. Consistency is achieved by block so that parcels in the same block are assigned to the same model.

The following section discusses the spatial lag framework used to address heterogeneity across property characteristics. The next section discusses the tests for vertical equity and quantile regressions. The final section summarizes the method and conclusions.

Spatial Lag Model Framework

Consider estimating property values for single-family and multifamily homes in the borough of Brooklyn, New York. For the purpose of assessment, single-family and multifamily homes with two or three residential units are considered in the same tax class, are assessed similarly, and are given the same tax rate. Estimating property value in terms of prices of sold parcels is specified by a general equation in natural log form (the log form is used in price regressions since it guarantees a positive price):

(1)

$$\ln Price = \ln\beta_0 + \beta_1 \ln Z_1 + \ldots + \beta_s \ln Z_s + \Sigma_{i=1}^{n}\gamma_i X_i + \Sigma_{j=1}^{m}\delta_j D_j + \varepsilon.$$

The coefficients $\beta_1 \ldots \beta_s$ are elasticities, and therefore the variables $Z_1 \ldots Z_s$ are continuous variables typically associated with size (house size, land size, and the like). The n coefficients γ measure the growth in price for a unit change in the variable X and are associated with other continuous variables such as age or distance. The m coefficients δ are adjustment factors based on the dummy variables D (e.g., categorical variables of quality and style, regular versus irregular shaped lot, zoning, and so on). The error ε is assumed to be

$$iid(0, \sigma_\varepsilon^2 I).$$

When the assumption on the errors fail due to location (e.g., errors showing a pattern by area), then heterogeneity across locations has not been fully modeled. It could be that delineating the model by area is insufficient and that interactions across areas need to be taken into account. The spatial lag model can be used to address such a scenario. Spatial autocorrelation occurs when prices in one location are correlated with prices in neighboring locations (e.g., several neighborhoods can have the spillover effect of employment growing in one neighborhood). With spatial autocorrelation in the data, model estimation using equation 1 generates inconsistent estimates. Instead, a spatial lag term is added. Take a weighted average of neighboring locations (weighed by distance), denote it by *Wy*, and insert it in equation 1 giving:

(2)

$$\ln Price = \ln \beta_0 + \alpha Wy + \beta_1 \ln Z_1 + \ldots + \beta_s \ln Z_s + \Sigma_{i=1}^{n} \gamma_i X_i + \Sigma_{j=1}^{m} \delta_j D_j + u$$

where

α = the coefficient on the spatially lagged variable

W = the weight matrix

$u = iid(0, \sigma_u^2 I)$.

The term *Wy* is used to interpolate prices to unsold parcels. Several methods are available for calculating weighted price. In the standard definition of spatial lags, the form of correlation is known and specified in the weight matrix *W* (e.g., adjacent neighbors are given the same weight). Alternatively, *Wy* can also be substituted with other measures that incorporate information on neighboring prices, such as estimates from inverse weighted distance (IDW), nonparametric kernel, spline, and kriging. This article uses the spline method because it is simple to implement with any statistical package such as SAS or SPSS, and it performs well when the price surface is well populated.

Data

In estimating equation 2 for Brooklyn, the dataset consists of 4,310 parcels sold between first quarter 2010 and second quarter 2012. Sales data are compiled by the Department of Finance and are published on its website. Data were cleaned for non-arm's-length transactions: in particular, foreclosure sales, sales in which one party was a public entity, sales in which one party was a financial institution, sales indicating a transfer between relatives, and sales which transferred more than once within a year.

Time Trend Adjustment

Because sales occurred during different periods, a trend must be accounted for before the price surface is estimated. A time trend regression of the log of price per square foot on quarterly dummies was run to detrend the data to end of period:

$$\ln psf = \alpha + \beta_i \times \sum_{i=1}^{9} qt_i + \varepsilon,$$

where *qt* are the time dummies excluding the second quarter 2012 as the base period. The median price per square foot ($/sq ft) and the time-adjusted median are plotted in figure 1. The circles are the actual median price ($/sq ft) across time, while the dashed line shows that the *time-adjusted* median price ($/sq ft) is now stable or detrended around the median of $271.90 for the base period. Time-adjusted sale prices (TASP) rather than actual sale price are used in the models.

Spatial Regression: Correcting for Heteroskedasticity

Figure 2, a map of the TASP ($/sq ft), shows that there are a sufficient number of sales spread out in areas that are primarily single-family homes. A stepwise regression was run to determine the significant physical characteristics. The results of the stepwise regressions are given in table 1.

The first regression is labeled spatial regression. The dependent variable is the log of TASP ($/sq ft) (lnpricepsfadj); the independent variables are as follows:

- Predicted price from the spline surface estimation of sold parcels (P_lnpricepsfadj)
- Log of square footage of living area (lnsfla)
- Log of parcel frontage (lnprc_frontage) and parcel depth (lnprc_depth)

Figure 1. Median for actual and time-adjusted sale prices ($/sq ft) detrended to second quarter 2012 level

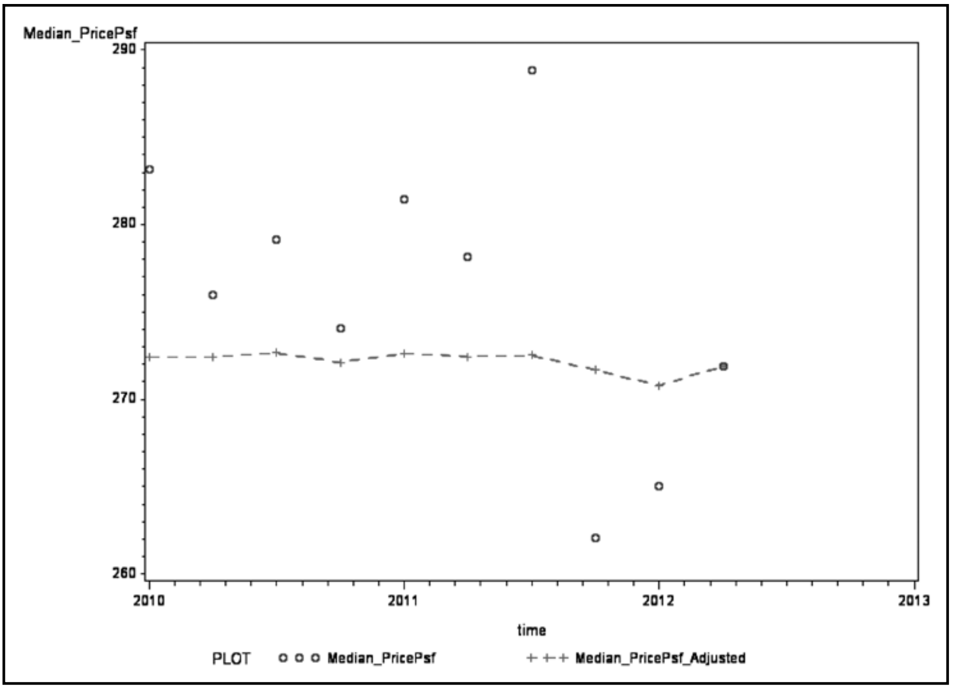


Figure 2. Spatial distribution of time-adjusted sale price ($/sq ft) in Brooklyn, New York

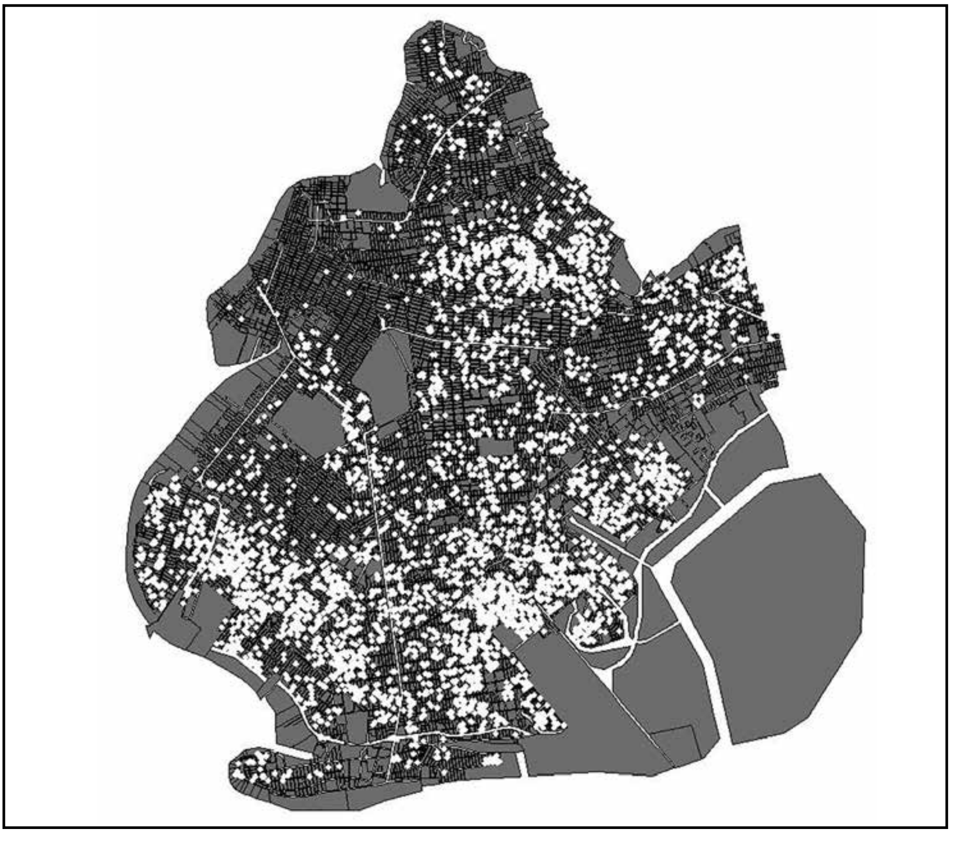


Table 1. Regression models

Spatial Regression					Multiple Regression Analysis				
Dependent Variable = lnpricepsfadj	**lnpricepsfadj**			**Statistical Result**	**Dependent Variable = lnpricepsfadj**	**lnpricepsfadj**			**Statistical Result**
Median ratio	0.9971				**Median ratio**	0.9938			
Adj R^2	0.8306				**Adj R^2**	0.5072			
COD	0.1160			*satisfies IAAO*	**COD**	0.2016			*does not satisfy IAAO*
PRD	1.0213			*satisfies IAAO*	**PRD**	1.0623			*does not satisfy IAAO*
White test	**DF**	**Chi-Square**	**Pr > ChiSq**		**White test**	**DF**	**Chi-Square**	**Pr > ChiSq**	
	240	233.54	0.6053	*Homoskedastic*		273	361.8	0.0003	*Heteroskedastic*
Variable	**Parameter Estimate**	***t* Value**	**Pr > |t|**		**Variable**	**Parameter Estimate**	***t* Value**	**Pr > |t|**	
Intercept	2.5739	21.27	<.0001		Intercept	9.49223	64.18	<.0001	
P_lnpricepsfadj	0.9135	91.56	<.0001		lnsfla	−0.54659	−21.67	<.0001	
lnsfla	−0.51639	−42.69	<.0001		lnprc_frontage	0.11788	5.07	<.0001	
lnprc_frontage	0.11434	7.83	<.0001		lnbld_depth	0.07611	3.26	0.0011	
lnprc_depth	0.13874	8.95	<.0001		lnraildist	−0.03677	−3.85	0.0001	
lnbld_frontage	0.11704	4.86	<.0001		BuiltFAR	−0.15213	−7.6	<.0001	
lnbld_depth	0.10688	7.5	<.0001		maxfar	−0.19639	−8.14	<.0001	
maxfar	0.08159	5.65	<.0001		maxfar2	0.03203	5.13	<.0001	
maxfar2	−0.01528	−4.14	<.0001		altage	0.00037495	2.16	0.0307	
lnaltage	−0.03969	−4.03	<.0001		gar_sqft	0.00022353	4.41	<.0001	
altage	0.00113	4.35	<.0001		stories	0.07642	7.18	<.0001	
gar_sqft	0.00013938	4.55	<.0001		raildist	−0.00000912	−2.7	0.0069	
stories	0.06068	9.57	<.0001		basement_dummy	0.12363	4.24	<.0001	
raildist	0.00000941	−8.58	<.0001		multifamily	−0.01824	−1.77	0.0772	
basement	0.06937	4.07	<.0001		brick	0.03598	3.73	0.0002	
multifamily	0.01628	2.66	0.0078		masonry_artf	0.09607	2.55	0.0109	
alum_vinyl	−0.02469	−3.62	0.0003		masonry_comb	0.0361	1.64	0.1001	
composition	−0.01709	−2.46	0.0138		composition	−0.03629	−2.94	0.0033	
attached	−0.01493	−2.09	0.0363		stucco	0.07006	4.45	<.0001	
row	−0.07417	−7.97	<.0001		wood	−0.043	−1.75	0.0796	
conventional	−0.06203	−4.52	<.0001		attached	−0.02468	−2.04	0.0413	
old_style	−0.07916	−8.74	<.0001		row	−0.07871	−4.99	<.0001	
					conventional	−0.09265	−3.96	<.0001	
					old_style	−0.03322	−2.15	0.0312	

- Log of building frontage (lnbld_frontage) and building depth (lnbld_depth)
- Maximum floor area ratio (maxfar, maxfar2 for its square)
- Age since last altered (altage and lnaltage), garage square footage (gar_sqft), number of stories (stories)
- Distance to the nearest subway station (raildist)
- Dummy variables of whether the property has a basement (basement), whether it is a single-family or multifamily residence, style (attached, row, conventional, old_style), and exterior construction (alum_vinyl, composition).

The median ratio is the predicted price from the regression divided by TASP. In a good model, the median ratio is close to 1 as with this model's ratio of 0.9971. The adjusted R^2 shows that the regression explains 83.06 percent of the variation in price. The COD is used to measure uniformity in assessments. Thus the COD measures the average absolute deviation of the individual sales ratios around the median ratio (for this analysis, the assessment ratio is taken to be the sales ratio of predicted price divided by TASP). A high COD suggests a lack of uniformity among individual assessments. For residential properties, the maximum allowable COD is 15 percent. This spatial regression has a COD of 11.6 percent, which satisfies IAAO standards.

The White test statistically finds homoskedasticity in the residuals of the spatial regression. This means that the variance of the errors does not depend on the independent variables of the model; in other words, errors are not more volatile as square footage increases, for example. The consequence of homoskedasticity in the residuals is that statistics that depend on the error variance are valid. This includes confidence intervals for the regression *t* statistics and the COD.

Contrast the results of spatial regression analysis with multiple regression analysis (MRA). MRA is a stepwise regression with the spatial lag term removed. The adjusted R^2 dropped to 50.72 percent, and the COD increased to 20.16 percent. More importantly, the residuals are heteroskedastic, which affects inference on the *t* statistics and the COD.

Table 1 illustrates the importance of taking spatial correlation into consideration. Compared to MRA, the addition of the spatial lag corrected for heteroskedasticity, significantly improved the regression fit, and decreased the COD to conform to IAAO standards.

Vertical Equity

Horizontal equity has been satisfied, as indicated by homoskedasticity in the residuals and a COD within IAAO standards. Now consider whether the spatial lag regression satisfies vertical equity. (Gloudemans [1999, chapters 5 and 60] discusses other tests for horizontal equity. This article uses the White test for homoskedasticity and the COD range as a test for horizontal equity.) Table 1 shows a PRD of 1.0213, which falls within the IAAO bounds. However, the bound of 0.98 and 1.03 are affected by how much assessment ratios vary with price (similar to how the presence of heteroskedasticity affects the width of confidence intervals). Testing for vertical equity and correcting for it are discussed in this section.

The test for vertical assessment equity is a regression of the assessment ratio (in this case the sales ratio) on sale price. The regression is

(3)

$$Assessment\_ratio = \beta_1 + \beta_2 \times TASP + v.$$

The test is the *t* statistic for the null hypothesis that $\beta_2 = 0$. A nonsignificant *t* statistic means assessment equity is satisfied since the assessment ratio does not depend on the price level.

Tests of Vertical Equity

The White test for heteroskedasticity considers error volatility with respect to the *independent* variables of the model. The PRD is affected by movement of error with respect to the sale price (the *dependent* variable). The IAAO standard is that the PRD falls in the range of 0.98–1.03. For the PRD to be valid, the regression (equation 3) must first be validated to ensure that the sales ratio does not move with price. If the *t* statistic is not significant, then the PRD can be used to gauge vertical assessment equity.

The importance of doing the regression test first is illustrated in this model. The PRD is 1.0213 in table 1, which means it satisfies the IAAO standard of vertical assessment equity. Figure 3 shows the graph of sales ratio versus TASP; table 2 shows the regression test of assessment equity. Clearly it is indicative of regressive assessment since the sales ratio falls when the price level increases. The correlation is −44 percent and the regression shows a *t* statistic of −26.11, which is significant.

Quantile Regressions

This spatial model does not support vertical equity, as indicated by a significant relationship between assessment ratio and price. I discuss a methodology that aims to correct for assessment inequity by reducing the correlation between assessment ratio and price.

Figure 3. Sales ratio versus time-adjusted sale price

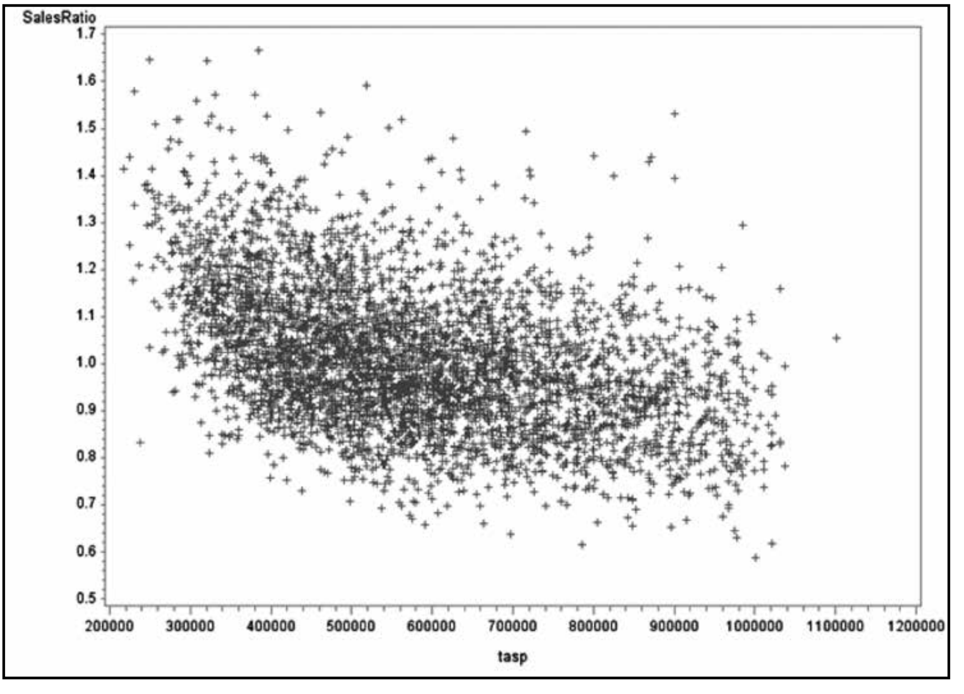


Table 2. Regression test of assessment equity

Dependent Variable =	SalesRatio			Statistical Result
Adj R^2	0.1938			
Correlation*	−0.44048			
Variable	**Parameter Estimate**	***t* Value**	**Pr > |*t*|**	
Intercept	1.2029	139.7	<.0001	
TASP	−3.70E−07	−26.11	<.0001	*Assessment Inequity*

**Correlation between Assessment Ratio (SalesRatio) and TASP*

Quantile regression is a type of regression analysis. Whereas the method of least squares estimates the conditional mean of the response variable, quantile regression estimates the conditional quantile of the response variable. For example, if the quantile specified is the median, then the regression estimates the conditional median of the response variable. Alternatively, while the method of least squares sets the average errors to zero, median regression would set the median error to zero.

One advantage of quantile regression over least squares is that the method is more robust to outliers. However, the main attraction of quantile regression is that it provides a more comprehensive analysis of the relationship of the response and explanatory variables in terms of both central tendency and dispersion. Note that quantile regression uses the full sample in estimating each quantile relation. It does not break the sample into different strata, so small sample problems are not an issue.

Figure 4 graphs the log of TASP on the log of assessment ratios. The lines are the quantile regression estimates of the equation

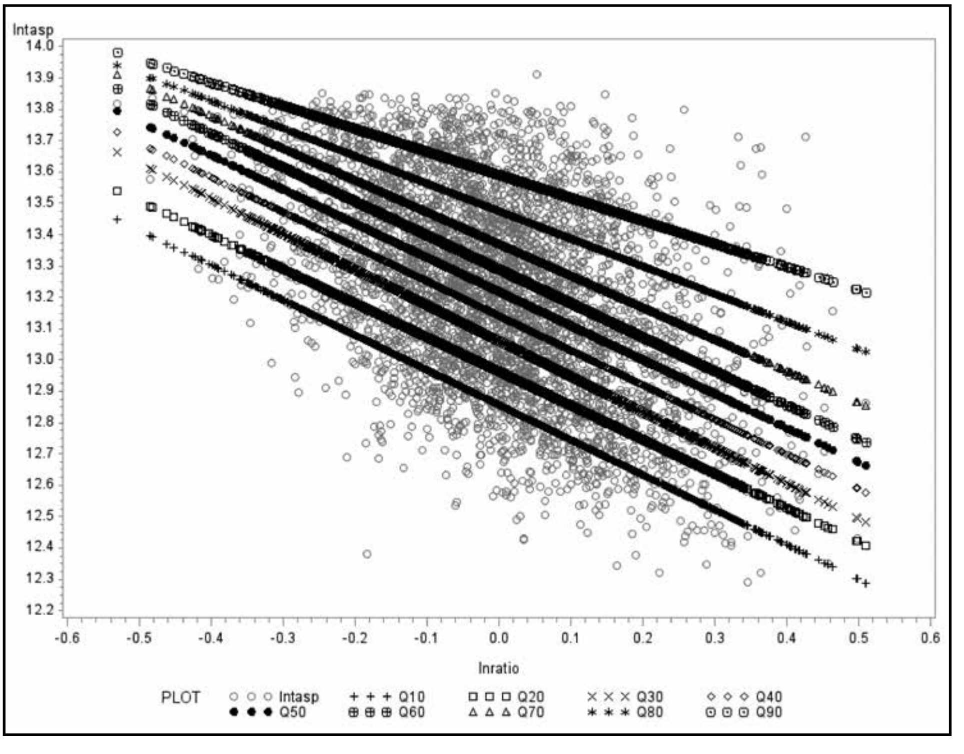
(4)

$$\ln(TASP) = \beta_1 + \beta_2 \times \ln(Assessment\_ratio) + v.$$

For example, the median regression is given by the line labeled Q50. The line labeled Q90 is the quantile regression line at the 90th percentile. Note that the response variable was reversed since it is the percentiles of TASP that are being modeled, not the percentiles of the assessment ratio.

Figure 4 shows the downward slope indicating the negative relation between price and assessment ratio (prices are higher for lower assessment ratios). The distance between the lowest and highest percentiles, the 10th and 90th percentiles, shows the unequal variance of price for different assessment ratios. Lower assessment ratios closer to the 10th percentile have a lower spread in prices compared to assessment ratios at the higher 90th percentile. Thus both the central tendency

Figure 4. Quantile regressions of sales ratio versus time-adjusted sale price (regressions in logs)



and variance are not constant between price level and assessment ratios.

This nonconstant relationship of price and assessment ratio is also shown in the different parameter estimates across percentiles. This is shown in figure 5. While the intercept is relatively constant across percentiles, the slope coefficient varies significantly.

Spatial-Quantile Regressions: Correcting for Vertical Inequity

To correct for vertical inequity, different quantile regressions are used for different price levels. However, in assigning parcels to a quantile model, price cannot be used as the assignment variable since mass appraisal requires unsold parcels to be valued (unsold parcels have no price information). Thus, the spatially interpolated price is used as the model assignment variable, which is defined for sold and unsold parcels. This is the explanatory variable labeled P_lnpricepsfadj in this regression model (exponentiated so assignment is by price not log of price).

Based on figure 5, three models were chosen at intervals where the slope coefficient changes most: at the 25th, 50th, and 70th percentiles. Thus the procedure of assignment is as follows:

1. Find the percentile distribution of the prices of *sold* parcels.
2. Interpolate the price surface of sold parcels. Unsold parcels can be derived from this surface by mapping their *x*–*y* coordinates on the estimated surface.
3. Run the quantile regression models at chosen points; in this case, the models are run at the 25th, 50th, and 70th percentiles. These are shown in table 3. The explanatory variables are first chosen from a stepwise regression, which was done in the previous section.
4. Assign parcels into quantile models according to the following rules:
   - If the interpolated price falls below the 25th percentile of the sold price distribution, assign it to regression estimated at the 25th percentile.
   - If the interpolated price falls between the 25th and 70th percentile of the sold price distribution, assign it to the median regression.

Figure 5. Estimated parameter by quantile for Log of TASP (lntasp)

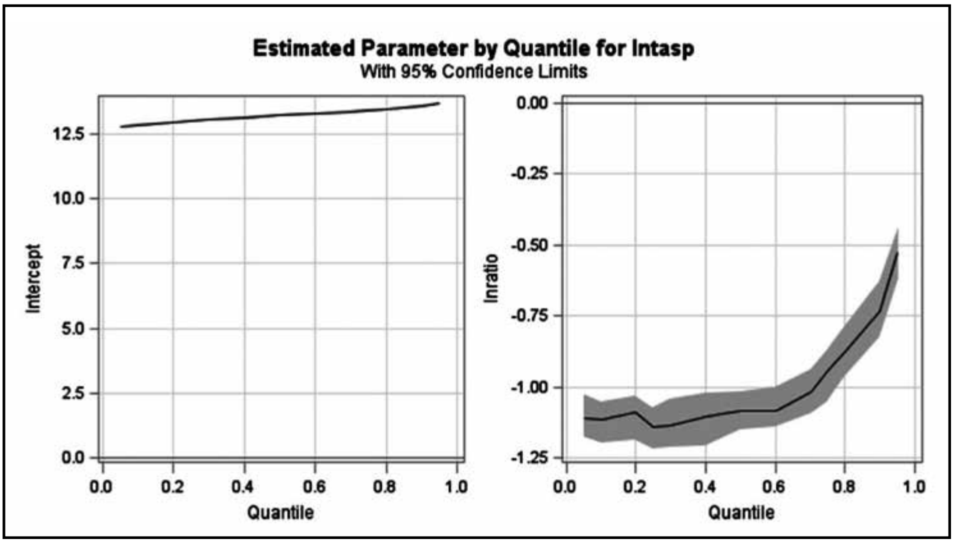


Table 3. Quantile regressions

Dependent Variable = lnpricepsfadj									
Variable	25th Quantile Model			50th Quantile Model			70th Quantile Model		
	Parameter Estimate	*t* Value	Pr > |t|	Parameter Estimate	*t* Value	Pr > |t|	Parameter Estimate	*t* Value	Pr > |t|
Intercept	2.7358	14.58	<.0001	2.6045	17.12	<.0001	2.697	17.14	<.0001
P_lnpricepsfadj	0.9247	75.3	<.0001	0.9159	79.59	<.0001	0.8877	70.55	<.0001
lnsfla	−0.5062	−36.85	<.0001	−0.5212	−29.84	<.0001	−0.5486	−32.39	<.0001
lnprc_frontage	0.1026	5.71	<.0001	0.1156	5.35	<.0001	0.1471	7.37	<.0001
lnprc_depth	0.102	4.33	<.0001	0.1216	6.17	<.0001	0.1484	7.93	<.0001
lnbld_frontage	0.0844	3.06	0.0022	0.1175	3.69	0.0002	0.146	4.54	<.0001
lnbld_depth	0.0884	5.52	<.0001	0.1102	6.34	<.0001	0.1277	7.34	<.0001
maxfar	0.0623	3.07	0.0022	0.0711	3.35	0.0008	0.0786	3.93	<.0001
maxfar2	−0.0131	−2.41	0.016	−0.012	−2.08	0.0376	−0.0126	−2.3	0.0216
lnaltage	−0.0396	−2.65	0.0081	−0.021	−1.97	0.049	−0.0383	−3.03	0.0024
altage	0.0009	2.4	0.0162	0.0006	1.71	0.0869	0.0011	3.4	0.0007
gar_sqft	0.0002	4.53	<.0001	0.0001	2.64	0.0083	0.0001	3.29	0.001
stories	0.0678	8.03	<.0001	0.0714	8.66	<.0001	0.0663	7.97	<.0001
raildist	−0.000010696	−6.82	<.0001	−1.06E-05	−7.56	<.0001	−0.000010372	−9.54	<.0001
basement	0.0701	3.1	0.002	0.0934	4.43	<.0001	0.0939	3.16	0.0016
multifamily	0.033	4.03	<.0001	0.0195	2.54	0.0111	0.0113	1.52	0.1296
alum_vinyl	−0.0278	−3.42	0.0006	−0.0262	−3.15	0.0016	−0.0246	−3.37	0.0008
composition	−0.0338	−2.84	0.0045	−0.0145	−1.62	0.1051	−0.0078	−0.93	0.3502
attached	−0.0043	−0.48	0.6343	−0.0169	−1.97	0.0487	−0.0084	−0.9	0.3666
row	−0.0949	−6.68	<.0001	−0.0884	−8.13	<.0001	−0.0826	−6.37	<.0001
conventional	−0.0819	−3.75	0.0002	−0.0685	−4.6	<.0001	−0.0647	−3.41	0.0007
old_style	−0.0828	−7.24	<.0001	−0.0932	−7.72	<.0001	−0.0901	−7.66	<.0001

- If the interpolated price falls above the 70th percentile of the sold price distribution, assign it to the regression estimated at the 70th percentile.

Note that price interpolation is done by using block coordinates so that two parcels in the same block are assigned to the same model. The procedure can be modified to interpolate the price surface on groups of blocks that define an area for more consistency across parcels.

Vertical Equity in Spatial-Quantile Models

This procedure is called the spatial-quantile method because it uses spatial estimates as assignments into levels of quantile regression models. Vertical equity is easily achieved in this framework by the choice and assignment into the models. Figure 6 and table 4 show the IAAO test of assessment equity. The sales ratio is flat to price level increases using the spatial-quantile method. The regression shows a *t* statistic of −2.37, which is not significant at the 1 percent level. Though still significant at the 5 percent level, the reduction in the relationship between assessment ratio and price is evident in the correlation coefficient dropping from 44.05 percent (absolute value) in the spatial lag model to 8.22 percent (absolute value) in the spatial-quantile model.

A summary comparison between the spatial and spatial-quantile regression models is given in table 5. While the spatial-quantile model has a slightly lower adjusted R^2 and higher COD compared to the spatial model (but within IAAO bounds), its advantage is it

Figure 6. Comparison of vertical equity between models

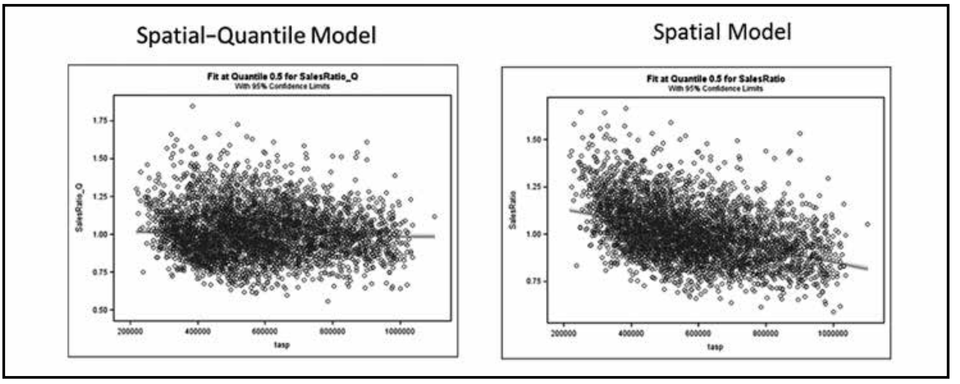


Table 4. Tests of vertical equity for spatial–quantile model and spatial model

Spatial-Quantile model

Dependent Variable =	SalesRatio_Q*			Statistical Result
PRD	1.0208			*satisfies IAAO*
Correlation	−0.0822			
Variable	**Parameter Estimate**	***t* Value**	**Pr > |t|**	
Intercept	1.0237	102.3	<.0001	
TASP	−3.75E−08	−2.37	0.0178	*Assessment Equity*

**Sales Ratio constructed from quantile models*

Spatial model

Dependent Variable =	SalesRatio			Statistical Result
PRD	1.0213			*affected by significant* t *statistic*
Correlation	−0.44048			
Variable	**Parameter Estimate**	***t* Value**	**Pr > |t|**	
Intercept	1.2029	139.7	<.0001	
TASP	−3.51E−07	−26.11	<.0001	*Assessment Inequity*

Table 5. Summary comparison between models

Model	MRA	Statistical Result	Spatial	Statistical Result	Spatial-Quantile	Statistical Result
Median ratio	0.9974		0.9971		1.00063	
Adj R^2	0.5072		0.8306		0.805	
COD	0.2016	*does not satisfy IAAO*	0.1160	*satisfies IAAO*	0.1246	*satisfies IAAO*
White test						
Pr > ChiSq	0.0003	*Heteroskedastic*	0.6053	*Homoskedastic*	0.1465	*Homoskedastic*
Correlation*	−75.74%		−44.05%		−8.22%	
Equity Test						
t value	−32.2	*Assessment Inequity*	−26.11	*Assessment Inequity*	−2.37	*Assessment Equity*
PRD	1.0623	*affected by significant* t *value*	1.0213	*affected by significant* t *value*	1.02082	*satisfies IAAO*

**Correlation of Assessment Ratio and TASP* improves vertical equity. (In this article, the 25th, 50th, and 70th percentiles were selected as the quantile models. An extension of this approach is to iterate over the choice of percentiles such that the COD is minimized, the R^2 is maximized, and the PRD is minimized to 1. This is a subject for future research.) The spatial-quantile approach significantly flattens the assessment ratio across price levels while conforming to IAAO standards.

Summary

In summary, this article introduces a method of improving vertical equity by incorporating levels of quantile regression in the prediction of prices. It is a two-step procedure. First, a spatial lag model addresses horizontal equity by improving the COD and correcting for heteroskedasticity. While other methods exist to address heteroskedasticity, the use of the spatial lag term also provides for an estimate of the price surface, which is used in the second step. The second step is the spatial-quantile procedure, which takes the spatial estimates of price (the spatial lag term exponentiated) and assigns them into quantile models. The contribution of this article is to illustrate that the spatial-quantile approach corrects for both horizontal and vertical equity with fit measures that conform to IAAO, even in a highly heterogeneous market like New York City.

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